AIME , AMC 12 Problems on Logarithms 2020 to 2023

Points A and B lie on the graph of $y = \log_2 x$. The midpoint of $\overline{AB}$ is $(6, 2)$. What is the positive difference between the x-coordinates of A and B?

$\textbf{(A)}\ 2\sqrt{11} \qquad \textbf{(B)}\ 4\sqrt{3} \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 4\sqrt{5} \qquad \textbf{(E)}\ 9$

A right rectangular prism whose surface area and volume are numerically equal has edge lengths $\log_2 x, \log_3 x,$ and $\log_4 x$. What is x?

$\textbf{(A)}\ 2\sqrt{6} \qquad \textbf{(B)}\ 6\sqrt{6} \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 576$

For how many integers n does the expression

$$\sqrt{\frac{\log(n^2) - (\log n)^2}{\log n - 3}}$$

represent a real number, where log denotes the base 10 logarithm?

$\textbf{(A)}\ 900 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 902 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 901$

What is the value of

$$(\log 5)^3 + (\log 20)^3 + (\log 8)(\log 0.25)$$

where $\log$ denotes the base-ten logarithm?

$\textbf{(A)}\ \frac{3}{2} \qquad \textbf{(B)}\ \frac{7}{4} \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ \frac{9}{4} \qquad \textbf{(E)}\ 3$

What is the value of

$$\frac{\log_2 80}{\log_{40} 2} - \frac{\log_2 160}{\log_{20} 2}?$$

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ \frac{5}{4} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \log_2 5$

What is the value of

$$\left(\sum_{k=1}^{20} \log_{5^k} 3^{k^2}\right) \cdot \left(\sum_{k=1}^{100} \log_{9^k} 25^k\right)?$$

$\textbf{(A)}\ 21 \qquad \textbf{(B)}\ 100\log_5 3 \qquad \textbf{(C)}\ 200\log_3 5 \qquad \textbf{(D)}\ 2{,}200 \qquad \textbf{(E)}\ 21{,}000$

What is the product of all the solutions to the equation

$$\log_{7x} 2023 \cdot \log_{289x} 2023 = \log_{2023x} 2023?$$

$\textbf{(A)}\ (\log_{2023} 7 \cdot \log_{2023} 289)^2 \qquad \textbf{(B)}\ \log_{2023} 7 \cdot \log_{2023} 289 \qquad \textbf{(C)}\ 1$

$\textbf{(D)}\ \log_7 2023 \cdot \log_{289} 2023 \qquad \textbf{(E)}\ (\log_7 2023 \cdot \log_{289} 2023)^2$

Positive real numbers $b \neq 1$ and n satisfy the equations

$$\sqrt{\log_b n} = \log_b \sqrt{n} \qquad \text{and} \qquad b \cdot \log_b n = \log_b(bn).$$

The value of n is $\frac{j}{k}$, where j and k are relatively prime positive integers. Find $j + k$.

There is a positive real number x not equal to either $\frac{1}{20}$ or $\frac{1}{2}$ such that

$$\log_{20x}(22x) = \log_{2x}(202x).$$

The value $\log_{20x}(22x)$ can be written as $\log_{10}(\frac{m}{n})$, where m and n are relatively prime positive integers. Find $m + n$.

There is a unique positive real number x such that the three numbers $\log_8(2x), \log_4 x,$ and $\log_2 x$, in that order, form a geometric progression with positive common ratio. The number x can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

The value of x that satisfies $\log_{2^x} 3^{20} = \log_{2^{x+3}} 3^{2020}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Determine all real values of x for which

$$\sqrt{\log_2 x \cdot \log_2(4x) + 1} + \sqrt{\log_2 x \cdot \log_2\left(\frac{x}{64}\right) + 9} = 4$$

(b) Suppose that $f(a) = 2a^2 - 3a + 1$ for all real numbers a and $g(b) = \log_{\frac{1}{2}} b$ for all $b > 0$. Determine all θ with $0 \leq \theta \leq 2\pi$ for which $f(g(\sin\theta)) = 0$.